

Backreaction and the Parametric Resonance of Cosmological Fluctuations

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We study the class of two-field inflationary universe models $\lambda\phi^4/4 + g^2\chi^2\phi^2/2$, in which parametric resonance during the initial stages of reheating can lead to an exponential amplification of the amplitude of cosmological fluctuations. Employing both analytical arguments and numerical simulations, we determine the time at which backreaction of fluctuations on the background fields shuts off the exponential growth, making use of the Hartree approximation, and including scalar metric perturbations. For the case $g^2/\lambda = 2$, we find that the amplitude of fluctuations after preheating will exceed the observational upper bound independent of the value of λ , unless the duration of inflation is very long. Cosmological fluctuations are acceptably small for $g^2/\lambda \geq 8$. We also find that the addition of χ -field self-interaction can limit the growth of fluctuations, and in the negative-coupling case the system can become effectively single-field, removing the resonance.

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I. INTRODUCTION

It has long been realized that reheating is a crucial part of the inflationary scenario. During reheating the large energy density contained within the coherently oscillating inflaton field is converted into particle excitations of whatever fields are coupled to the inflaton, vastly increasing the temperature and entropy density and setting the stage for the standard big bang phase. If inflation is ever to be a useful picture for describing the early universe, then it is essential to understand the details of how the vacuum energy is transformed into familiar particles.

Reheating can occur very efficiently through the process of parametric resonance [1–3]. Field modes within certain resonance bands in k -space grow exponentially with time, defining the “preheating” era. The possibility of resonant growth of linear scalar *metric* perturbations was first studied in [4]. Recently it has been argued that the resonance of scalar metric perturbations can extend to $k \ll aH$, *i.e.* that super-Hubble perturbations can be amplified [5,6]. This opens up the possibility of new observational consequences, since the scales relevant to the cosmic microwave background and large-scale structure are much larger than the Hubble radius during preheating. The importance of the gauge-invariant formalism for cosmological perturbations [7] and the study of the “traditionally conserved” Bardeen parameter ζ was emphasized in [8], where it was found that simple single-field chaotic inflation models do not exhibit super-Hubble growth beyond what is expected in the absence of parametric resonance. The absence of parametric amplification of super-Hubble modes in these single field models was shown to hold in a full nonlinear treatment [9], and a general no-go theorem in these models was suggested in [10].

For the first model which was claimed to exhibit

growth of super-Hubble metric perturbations beyond that of the usual theory of reheating [6] (see also [11]), it was soon realized that the growth was unimportant since it followed a period of exponential *damping* during inflation [12–14]. This damping of super-Hubble modes arises because the field perturbations which are amplified during preheating have an effective mass greater than the Hubble parameter during inflation. This results in a very “blue” power spectrum at the end of inflation, with a severe deficit at the largest scales [14]. The relatively plentiful small-scale modes can also grow resonantly during preheating. Thus the end of parametric resonance occurs when the backreaction of the dominant small-scale modes becomes important, and the cosmological-scale modes are still negligible. An obvious class of models to study, then, consists of those with small masses during inflation and strong super-Hubble resonance [15,16]. A simple example was provided by Bassett and Viniegra [17], namely that of a massless self-coupled inflaton ϕ coupled to another scalar field χ , *i.e.* a model with potential

$$V(\phi, \chi) = \frac{\lambda}{4}\phi^4 + \frac{g^2}{2}\phi^2\chi^2. \quad (1)$$

This model has been studied in detail, but in the absence of metric perturbations, by Greene *et al.* [18], who found that the model contains a strong resonance band for χ fluctuations which extends to $k = 0$ for the choice $g^2 = 2\lambda$. Bassett and Viniegra [17] found that super-Hubble metric perturbations are resonantly amplified as well in this model (see also [16]).

To date, however, none of the linearized analyses of parametric amplification of super-Hubble-scale metric fluctuations in the model (1) has included the effects of backreaction on the evolution of the fluctuations. The backreaction of the growing modes on the background

fields is expected to shut the growth down at some point, but exactly when? Backreaction is also the only hope to make models which exhibit parametric amplification of super-Hubble cosmological perturbations compatible with the Cosmic Background Explorer (COBE) normalization [19].

In this paper we will investigate the effects of backreaction on the growth of matter and metric fluctuations using the Bassett and Viniegra model (1) as our toy model. We will study the growth of scalar field and scalar metric perturbations, including the effect of backreaction in the Hartree approximation. We carefully treat the evolution during inflation, which can be very important for super-Hubble scales. We will compare the large-scale normalization predicted for this model with the COBE value, and find that, although backreaction is crucial in limiting the growth of the fluctuations, the final amplitude is larger than allowed by the COBE normalization (for supersymmetry-motivated coupling constant values), unless the period of inflation is very long. Note that the final amplitude of fluctuations in our model is independent of the scalar field coupling constant (unlike what happens without parametric resonance effects), but that it may depend on the duration of the inflationary period. We also extend the model to study the effect of χ -field self-coupling, which can be important in limiting the growth of fluctuations.

II. MODEL AND LINEARIZED DYNAMICS

A. Equations of motion and analytical theory

Our model is the two-real-scalar-field, gravitationally minimally coupled model specified by the Lagrangian density

$$\mathcal{L} = \sqrt{-g} \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{\lambda}{4} \phi^4 - \frac{g^2}{2} \phi^2 \chi^2 \right). \quad (2)$$

The field ϕ drives inflation, while χ is significant only after parametric resonance begins, so the inflationary dynamics is essentially that of $(\lambda/4)\phi^4$ chaotic inflation. Note that the behaviour of this system is expected to be robust under the addition of a small mass term $m_\phi^2 \phi^2$ with $m_\phi \ll \sqrt{\lambda} m_{\text{Pl}}$ and for the ratio of coupling constants satisfying $g/\sqrt{\lambda} < \sqrt{\lambda} m_{\text{Pl}}/m_\phi$ [18]. In particular, this will be the case for supersymmetric models, which motivate the choice $g^2 = 2\lambda$ [20]. In addition, we will show that large values of $g/\sqrt{\lambda}$ are in fact inconsistent with the significant amplification of super-Hubble modes. On the other hand, for $g/\sqrt{\lambda} > \sqrt{\lambda} m_{\text{Pl}}/m_\phi$, the theory of “stochastic resonance” for a massive inflaton may need to be applied [2].

It is traditional to separate the inflaton field into a homogeneous, “classical” background $\phi(t)$ and a perturbation $\delta\phi(x, t)$, which begins as sub-Hubble quantum vacuum fluctuations early in inflation. It will be useful to make a similar separation for the field χ , although we have no reason to expect a non-zero initial homogeneous χ component in this model. The equations of motion for the homogeneous parts of the inflaton and χ fields are the Klein-Gordon equations,

$$\ddot{\phi} + 3H\dot{\phi} + \lambda\phi^3 + g^2\phi\chi^2 = 0, \quad (3)$$

$$\ddot{\chi} + 3H\dot{\chi} + g^2\phi^2\chi = 0, \quad (4)$$

with Hubble parameter $H = \dot{a}/a$, where a is the scale factor. To complete the background dynamics we must specify the evolution of the background spacetime metric. We assume a spatially flat Friedmann-Robertson-Walker universe, for which the 0-0 Einstein equation gives the Friedmann equation

$$H^2 = \frac{8\pi}{3m_{\text{Pl}}^2} \left[\frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \dot{\chi}^2 + V(\phi, \chi) \right]. \quad (5)$$

For $\phi \gtrsim m_{\text{Pl}}$, the universe undergoes slow-roll inflation, with H approximately constant and the scale factor a increasing approximately exponentially with time. As slow-roll ends, the “damping” term $3H\dot{\phi}$ becomes less important in Eq. (3) and the field begins to oscillate about $\phi = 0$. This marks the start of the preheating period. Averaged over several oscillations, the equation of state (in the absence of backreaction) is very nearly that of a radiation-dominated universe [3], and the amplitude of the inflaton’s oscillations decays as a^{-1} . This is a consequence of the (near) conformal invariance of this massless model, which considerably simplifies the treatment of parametric resonance as compared with the massive case [18, 2].

In writing the linearized equations of motion for perturbations about the background, we will use the longitudinal gauge. For this model the metric can be written [7]

$$ds^2 = (1 - 2\Phi)dt^2 - a^2(t)(1 + 2\Phi)dx_i dx^i, \quad (6)$$

so scalar metric perturbations are described by the single variable Φ . The momentum-space first-order perturbed Einstein and Klein-Gordon equations are

$$\begin{aligned} 3H\dot{\Phi} + \left(\frac{k^2}{a^2} + 3H^2 \right) \Phi \\ = -\frac{4\pi}{m_{\text{Pl}}^2} \sum_i \left(\dot{\phi}_i \delta\dot{\phi}_i - \Phi \dot{\phi}_i^2 + V_{,i} \delta\phi_i \right), \end{aligned} \quad (7)$$

$$\dot{\Phi} + H\Phi = \frac{4\pi}{m_{\text{Pl}}^2} \sum_i \dot{\phi}_i \delta\phi_i, \quad (8)$$

$$\delta\ddot{\phi}_i + 3H\dot{\phi}_i + \frac{k^2}{a^2}\delta\phi_i + \sum_j V_{,ij}\delta\phi_j = 4\dot{\Phi}\dot{\phi}_i - 2V_{,i}\Phi, \quad (9)$$

where $\phi_1 \equiv \phi$, $\phi_2 \equiv \chi$, $V_{,i} \equiv \partial V/\partial\phi_i$, and comoving momentum k subscripts have been suppressed for clarity. Equations (7) and (8) can be combined to give

$$\Phi = \frac{\sum_i \left(\dot{\phi}_i \delta\phi_i + 3H\dot{\phi}_i \delta\phi_i + V_{,i}\delta\phi_i \right)}{-(m_{\text{Pl}}^2/4\pi)(k/a)^2 + \sum_i \dot{\phi}_i^2}, \quad (10)$$

which fixes Φ once the matter fields are known.

An important quantity in the study of the linear evolution of metric perturbations is the Bardeen parameter [21]

$$\zeta_k = \Phi_k - \frac{H}{\dot{H}} \left(\dot{\Phi}_k + H\Phi_k \right), \quad (11)$$

which for $k/a \ll H$ and *single field* models satisfies the “conservation law” [7]

$$(1+w)\dot{\zeta}_k = 0, \quad (12)$$

where $w = P/\rho$ is the equation of state (ρ and P denoting energy density and pressure, respectively). When $\dot{\Phi}_k$ can be neglected, Eqs. (11) and (12) can be combined to give the familiar result that the change in Φ_k on super-Hubble scales over some interval of time is determined solely by the change in equation of state.

In the absence of metric perturbations, the linearized dynamics in the model described above is known to exhibit parametric resonance during preheating [18]. To see this, it helps to take advantage of the near conformal invariance of the model and rewrite the equations in terms of conformally scaled fields $\tilde{\phi}_i \equiv a\phi_i$ and a dimensionless conformal time $x \equiv \sqrt{\lambda}\phi_0\eta$, where $\eta = \int dt/a$ and $\tilde{\phi}_0$ is the amplitude of inflaton oscillations at the start of preheating. Then, with the χ background and metric perturbations set to zero, the inflaton background and perturbed field equations (3) and (9) become

$$\tilde{\phi}'' + \frac{\tilde{\phi}^3}{\tilde{\phi}_0^2} = 0, \quad (13)$$

$$\delta\tilde{\phi}_k'' + \left(\kappa^2 + 3\frac{\tilde{\phi}^2}{\tilde{\phi}_0^2} \right) \delta\tilde{\phi}_k = 0, \quad (14)$$

$$\delta\tilde{\chi}_k'' + \left(\kappa^2 + \frac{g^2}{\lambda}\frac{\tilde{\phi}^2}{\tilde{\phi}_0^2} \right) \delta\tilde{\chi}_k = 0, \quad (15)$$

where $\kappa^2 \equiv k^2/(\lambda\tilde{\phi}_0^2)$ is a dimensionless comoving momentum and primes denote derivatives with respect to the scaled conformal time x . Here we have ignored terms proportional to a''/a since preheating is a nearly-radiation-dominated phase in this model [3]. The conformal field $\tilde{\phi}$ then undergoes constant amplitude elliptic

cosine oscillations, while the perturbation equations are Lamé equations [22], which are known to exhibit resonance within certain bands in parameter space [18]. In particular, the $\delta\tilde{\chi}_k$ equation exhibits strong resonance for a band that includes $k = 0$ for the supersymmetric point $g^2/\lambda = 2$, and weak “narrow” resonance in small-scale bands. On the other hand, $\delta\tilde{\phi}_k$ exhibits narrow resonance for a sub-Hubble momentum range, independent of the coupling constants. For resonant modes, the growth is a modulated exponential, $\delta\tilde{\chi}_k \propto e^{\mu_k x}$, with Floquet index μ_k . If we allow g^2/λ to vary, we find a sequence of $\delta\tilde{\chi}_k$ resonance bands for $k = 0$, centred at $g^2/\lambda = 2n^2$ with width $2n$, for positive integral n [18]. The Floquet index reaches a maximum value of $\mu_{\text{max}} \simeq 0.238$ at the centre of each $k = 0$ band.

B. Numerical results

For our numerical calculations, we were primarily interested in the behaviour of cosmological-scale matter and metric modes. Thus we evolved a scale which left the Hubble radius (at time t_0) at about $N = 50$ e -folds before the end of inflation. For $(\lambda/4)\phi^4$ models, the number of e -folds during slow-roll inflation after initial time t_0 is [23]

$$N \simeq \pi \left(\frac{\phi(t_0)}{m_{\text{Pl}}} \right)^2; \quad (16)$$

thus we used the homogeneous inflaton initial value of $\phi(t_0) = 4m_{\text{Pl}}$. We began the calculations with the modes still somewhat inside the Hubble radius, so the initial conditions for the matter field fluctuations were simply given by the conformal vacuum state

$$\delta\phi_{ik}(t_0) = \frac{1}{a^{3/2}(t_0)} \left(\frac{1}{2\omega_i(t_0)} \right)^{1/2}, \quad (17)$$

$$\dot{\delta\phi}_{ik}(t_0) = -i\omega_i(t_0)\delta\phi_{ik}(t_0), \quad (18)$$

with $\omega_\phi^2(t) = (k/a)^2 + 3\lambda\phi^2 + g^2\chi^2$ and $\omega_\chi^2(t) = (k/a)^2 + g^2\phi^2$. Physically, the $a^{-3/2}$ dependence arises because particle number densities $n_k \propto |\delta\phi_{ik}|^2$ must decay like a^{-3} in the massive, adiabatic regime. The initial metric perturbations were then determined by Eq. (10).

To illustrate the dynamics in the absence of backreaction, we numerically integrated the coupled set of background equations (3) and (5) and perturbation equations (8) and (9) using the initial conditions described above, and for $g^2/\lambda = 2$, $\lambda = 10^{-14}$, and a zero χ background. We used the constraint Eq. (10) as well as the conservation equation (12) to check the accuracy of the calculations. In Fig. 1 we display the evolution of our cosmological modes, together with the Bardeen parameter ζ_k , during inflation and preheating. For each of the perturbations $X_k = \delta\chi_k$, $\delta\phi_k$, Φ_k , and ζ_k we plot the power spectrum [24]

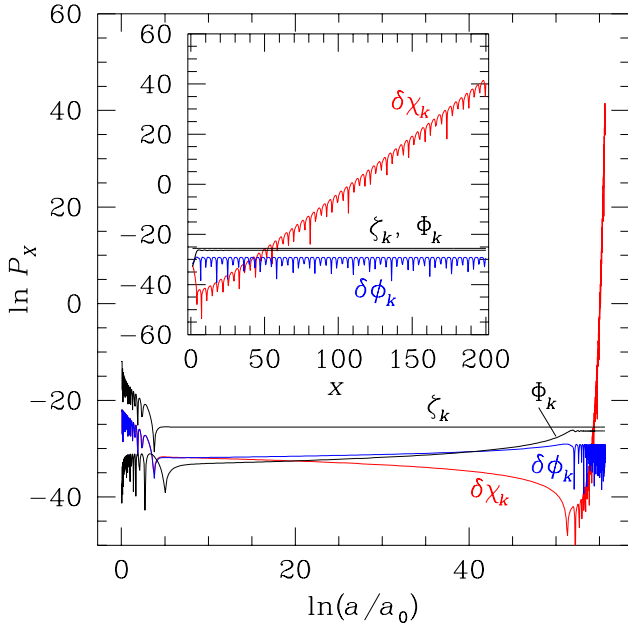


FIG. 1. Numerical simulation of linear cosmological-scale perturbations in the two field model described in the text, in the absence of backreaction. Plotted are the logarithms of the power spectra $\mathcal{P}_X(k) = (k^3/2\pi^2)|X_k|^2$, for $X_k = \delta\chi_k$, $\delta\phi_k$, Φ_k , and ζ_k , and using $m_{\text{Pl}} = 1$, $g^2/\lambda = 2$, $\lambda = 10^{-14}$, and zero χ background. The main figure shows the evolution during inflation, while the inset details the behaviour during preheating, using the rescaled conformal time x . This particular comoving scale leaves the Hubble radius approximately 5 e -folds after the start of the simulation.

$$\mathcal{P}_X(k) = \frac{k^3}{2\pi^2}|X_k|^2, \quad (19)$$

rather than the mode amplitudes, to facilitate comparison with the COBE measured normalization which gives $\mathcal{P}_\Phi \sim 10^{-10}$ [19].

Figure 1 shows how the modes begin early in inflation as sub-Hubble oscillations, and become “frozen in” after they exit the Hubble radius. Note that the $\delta\chi_k$ fluctuation experiences some damping late in inflation, when its effective mass squared $g^2\phi^2$ becomes somewhat greater than H^2 , which decreases like ϕ^4 in the slow-roll approximation. The inflaton perturbation $\delta\phi_k$, however, stays roughly constant during inflation even though its effective mass is comparable to that of the $\delta\chi_k$ mode. This is because of the coupling between $\delta\phi_k$ and Φ_k in the linearized perturbation equations. We can also observe a growth of Φ_k between the time the mode exits the Hubble radius and the beginning of preheating, by a factor of approximately 20, in good agreement with the growth predicted from the “conservation law” Eq.(12). Also note that after this small growth stage the cosmological-scale metric power spectrum ends up close to the 10^{-10} ($\sim e^{-23}$) level, as the standard theory predicts for $\lambda \simeq 10^{-14}$ in the absence of parametric resonance [7]. During preheating we observe exponential

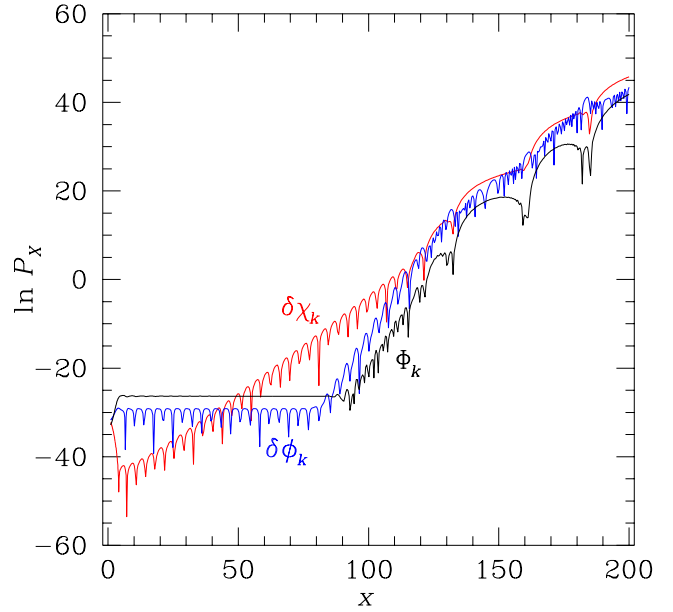


FIG. 2. Simulation of linear cosmological-scale perturbations in the absence of backreaction as in Fig. 1, but with a non-zero initial background $\chi(t_0) = 10^{-10} m_{\text{Pl}}$. Plotted are the logarithms of the power spectra $\mathcal{P}_X(k) = (k^3/2\pi^2)|X_k|^2$, for $X_k = \delta\chi_k$, $\delta\phi_k$, and Φ_k , and using $m_{\text{Pl}} = 1$, $g^2/\lambda = 2$, and $\lambda = 10^{-14}$. Coupling through the χ background causes $\delta\phi_k$ and Φ_k to grow initially at twice the rate of $\delta\chi_k$.

growth of $\delta\chi_k$ while the super-Hubble $\delta\phi_k$ mode does not grow, as expected from the analytical theory. Φ_k and ζ_k remain constant, since according to Eq.(8) the metric perturbations couple only to $\delta\phi_k$ in the absence of a χ background, at linear level [16].

To observe the effect of including a non-zero homogeneous χ background, we repeated the above calculation with an initial value of $\chi(t_0) = 10^{-10} m_{\text{Pl}}$ (this value illustrates well the various stages of evolution). Figure 2 indicates that $\delta\chi_k$ grows as before, but $\delta\phi_k$ and Φ_k now grow initially with twice the Floquet index of $\delta\chi_k$. This is the result of the driving term $2g^2\phi\chi\delta\chi_k$ in the equation of motion for $\delta\phi_k$, Eq.(9), which contains two factors growing like $e^{\mu_{\text{max}}x}$ (clearly the evolution of the background χ will be essentially the same as that of $\delta\chi_k$ for $k \ll aH$). Once the background χ field becomes comparable to the inflaton background, all the perturbations synchronize and grow at the same rate. We will discuss the significance of the homogeneous χ field in relation to the nonlinear evolution of the fields in the next section.

III. BACKREACTION

A. Equations of motion

The linearized equations in the previous section describe unbounded growth of perturbations during res-

onance. In reality this growth must of course stop at some point, namely when the perturbed field values are on the order of the background values. A full nonlinear simulation will include this effect automatically, but approximation methods can alleviate the computational costs significantly. A common approach to approximate this backreaction on the background and perturbation evolution is to include Hartree terms in the equations of motion [25,2]. This entails making the replacements $\phi_i \rightarrow \phi_i + \delta\phi_i$, $\delta\phi_i^2 \rightarrow \langle\delta\phi_i^2\rangle$, and $\delta\phi_i^3 \rightarrow 3\langle\delta\phi_i^2\rangle\delta\phi_i$. In this approximation, the background equations (3) – (5) become

$$H^2 = \frac{8\pi}{3m_{\text{Pl}}^2} \left[V(\phi, \chi) + \frac{3}{2}\lambda\phi^2\langle\delta\phi^2\rangle + \frac{g^2}{2}\phi^2\langle\delta\chi^2\rangle + \frac{g^2}{2}\chi^2\langle\delta\phi^2\rangle + \frac{1}{2}\sum_i \left(\dot{\phi}_i^2 + \langle\dot{\delta\phi}_i^2\rangle + \frac{1}{a^2}\langle\nabla\delta\phi_i^2\rangle \right) \right], \quad (20)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + 3\lambda\langle\delta\phi^2\rangle\phi + g^2\langle\delta\chi^2\rangle\phi = 0, \quad (21)$$

$$\ddot{\chi} + 3H\dot{\chi} + V_{,\chi} + g^2\langle\delta\phi^2\rangle\chi = 0. \quad (22)$$

Similarly, the momentum-space linearized field perturbation equations (9) become

$$\ddot{\delta\phi}_k + 3H\dot{\delta\phi}_k + \left(\frac{k^2}{a^2} + 3\lambda\langle\delta\phi^2\rangle + g^2\langle\delta\chi^2\rangle \right) \delta\phi_k + \sum_j V_{,\phi j} \delta\phi_{jk} = 4\dot{\phi}\dot{\Phi}_k - 2V_{,\phi}\Phi_k, \quad (23)$$

$$\ddot{\delta\chi}_k + 3H\dot{\delta\chi}_k + \left(\frac{k^2}{a^2} + g^2\langle\delta\phi^2\rangle \right) \delta\chi_k + \sum_j V_{,\chi j} \delta\phi_{jk} = 4\dot{\chi}\dot{\Phi}_k - 2V_{,\chi}\Phi_k. \quad (24)$$

In this approach, the field fluctuations are calculated self-consistently from the relations

$$\langle\delta\phi_i^2\rangle = \frac{1}{(2\pi)^3} \int d^3k |\delta\phi_{ik}|^2. \quad (25)$$

In practice, the resonance band will provide a natural ultraviolet cutoff.

The Hartree terms approximate the full nonlinear dynamics of the fields. To illustrate what this approximation entails, we may consider the exact dynamics of the fields, treating the Klein-Gordon equation as a classical field equation. This should be a good approximation soon after the beginning of the resonance stage, since occupation numbers will grow exponentially with time [2,18]. As an example, consider the exact evolution equation for

$\delta\phi$ in position space, obtained by perturbing the Klein-Gordon equation, setting the background χ to zero, and ignoring metric perturbations,

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} + \frac{1}{a^2}\nabla^2\delta\phi + 3\lambda\phi^2\delta\phi + 3\lambda\phi\delta\phi^2 + \lambda\delta\phi^3 + g^2\delta\chi^2\phi + g^2\delta\chi^2\delta\phi = 0. \quad (26)$$

The terms in this equation describing the interaction between the ϕ and χ fields become in momentum space

$$\frac{g^2\phi}{(2\pi)^{3/2}} \int d^3k' \delta\chi_{\mathbf{k}'} \delta\chi_{\mathbf{k}-\mathbf{k}'} + \frac{g^2}{(2\pi)^3} \int d^3k' d^3k'' \delta\chi_{\mathbf{k}'} \delta\chi_{\mathbf{k}''} \delta\phi_{\mathbf{k}-\mathbf{k}'-\mathbf{k}''}. \quad (27)$$

Thus the Hartree term $g^2\langle\delta\chi^2\rangle\delta\phi_k$ in Eq.(23) corresponds to the second term in expression (27), restricted to $\mathbf{k}'' = -\mathbf{k}'$. Physically, this means that only scattering events which do not change the $\delta\phi_k$ momentum are included in the Hartree approximation, and “rescattering” events are ignored.

It is important to notice that the first term in (27), which scatters particles from the homogeneous inflaton background into mode $\delta\phi_k$, could be larger than the Hartree term since initially $|\phi| > |\delta\phi|$, unless the first term vanishes upon averaging (integrating) over the entire phase space of contributing terms (which is what is assumed in the Hartree approximation). If it does not vanish, the first term in (27) will act as a driving term for the $\delta\phi$ modes in (26). Since some $\delta\chi$ modes experience parametric amplification with Floquet exponent μ , this term will lead to an important second-order effect, namely the growth of $\delta\phi$ as $e^{2\mu x}$. This effect is left out in the Hartree approximation. Because the metric perturbations are coupled to $\delta\phi$ through Eq. (8), we also expect that, with the homogeneous χ set to zero, the Hartree approximation will miss the corresponding growth of Φ . Note, however, that by including the χ background term $2g^2\chi\phi\delta\chi$ in (26), and setting $\chi^2 \sim \langle\delta\chi^2\rangle$, we can approximate the effect of the important first term in (27), as we saw in Fig. 2.

Just as with the scalar fields, metric fluctuations may grow rapidly in our model. We can account for the backreaction of metric perturbations through the effective energy-momentum tensor formalism of Abramo *et al.* [26]. This involves expanding the Einstein equations to second order in the perturbations and taking the spatial average to obtain effective background equations. In our case, the metric and inflaton equations (20) and (21) become, with background χ set to zero,

$$H^2 = \frac{8\pi}{3m_{\text{Pl}}^2} \left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{3}{2}\lambda\phi^2\langle\delta\phi^2\rangle + \frac{g^2}{2}\phi^2\langle\delta\chi^2\rangle + \frac{1}{2}\sum_i \left(\langle\dot{\delta\phi}_i^2\rangle + \frac{1}{a^2}\langle(\nabla\delta\phi_i)^2\rangle \right) + 2\lambda\phi^3\langle\Phi\delta\phi\rangle \right] + 4H\langle\Phi\dot{\Phi}\rangle - \langle\dot{\Phi}^2\rangle + \frac{3}{a^2}\langle(\nabla\Phi)^2\rangle, \quad (28)$$

$$\begin{aligned}
& (\ddot{\phi} + 3H\dot{\phi})(1 + 4\langle\Phi^2\rangle) + \lambda\phi^3 + 3\lambda\langle\delta\phi^2\rangle\phi \\
& + g^2\langle\delta\chi^2\rangle\phi - 2\langle\Phi\ddot{\phi}\rangle - 4\langle\dot{\Phi}\dot{\phi}\rangle \\
& - 6H\langle\Phi\dot{\phi}\rangle + 4\dot{\phi}\langle\dot{\Phi}\rangle - \frac{2}{a^2}\langle\Phi\nabla^2\delta\phi\rangle = 0. \quad (29)
\end{aligned}$$

B. Analytical estimates

1. Evolution of perturbations during inflation

Perturbations will grow during parametric resonance until backreaction becomes important. We can analytically estimate the amount of growth by estimating the time at which the Hartree term $g^2\langle\delta\chi^2\rangle$ is of the order of the background $\lambda\phi^2$ (cf. Eq. (23)). Note that in the absence of metric fluctuations, such an estimate should be accurate, at least for $g^2/\lambda \sim 1$, as nonlinear lattice simulations indicate [27,28]. In addition, we expect the matter sector to dominate the dynamics. In order to estimate the variance $\langle\delta\chi^2\rangle$, we will need to calculate the evolution of $\delta\chi_k$ modes, starting from the adiabatic vacuum inside the Hubble radius, continuing through inflation, and finally through preheating. The evolution during inflation is quite complicated, and will have a crucial effect on the final variances, so we will describe the inflationary stage in some detail. We consider general values of g^2/λ , rather than just the supersymmetric point.

We will only need to consider the contribution to $\langle\delta\chi^2\rangle$ from modes which are super-Hubble at the start of preheating. To see this, first note that for $g^2/\lambda = 2$, the small-scale boundary of the strongest (and largest-scale) resonance band is at $k_{\max}/a \simeq \sqrt{\lambda}\phi_0/2$, where $\phi_0(t)$ is the amplitude of inflaton oscillations during preheating [18]. Next, we can use the Friedmann equation (5) to write the Hubble parameter in terms of ϕ_0 , giving

$$H^2 = \frac{2\pi}{3m_{\text{Pl}}^2}\lambda\phi_0^4. \quad (30)$$

(Note that this equation also applies approximately during slow-roll.) Using the value $\phi_0 = 0.2m_{\text{Pl}}$, we calculate the ratio $aH/k_{\max} \simeq 0.6$ at the start of preheating. Thus the Hubble radius corresponds closely to the smallest resonant scale. This result is not very sensitive to g^2/λ as long as we are near the centre of a band, *i.e.* $g^2/\lambda = 2n^2$, since k_{\max} increases only slowly with g^2/λ in this case [18]. Also, we can ignore the resonance bands at higher k values, since they correspond to narrow resonance.

To estimate the evolution of $\delta\chi_k$ on super-Hubble scales during inflation, we can ignore terms containing the background χ as well as the spatial gradient term in Eq. (9), resulting in a damped harmonic oscillator equation with time-dependent coefficients,

$$\ddot{\delta\chi}_k + 3H\dot{\delta\chi}_k + g^2\phi^2\delta\chi_k = 0. \quad (31)$$

During slow-roll, we can use the adiabatic approximation to find solutions to this equation, since $|\dot{H}| \ll H^2$. Thus

for $g^2\phi^2 > (3H/2)^2$ we have underdamped oscillations with damping envelope

$$\delta\chi_k \propto \exp\left[-\int (3H/2)dt\right] = a^{-3/2}. \quad (32)$$

For $g^2\phi^2 < (3H/2)^2$, we have the overdamped case with two decaying modes. Ignoring the more rapidly decaying mode, we obtain

$$\delta\chi_k \propto \exp\left[-\int \left(3H/2 - \sqrt{9H^2/4 - g^2\phi^2}\right)dt\right]. \quad (33)$$

In this case, the fluctuations are very slowly decaying in the massless limit $g^2\phi^2 \ll (3H/2)^2$, while they approach the $a^{-3/2}$ decay as $g^2\phi^2 \rightarrow (3H/2)^2$.

During slow-roll we have $H^2 \propto \phi^4$ (see Eq. (30)), so that H^2 decreases more rapidly than $g^2\phi^2$, and there is a transition between the over- and underdamped stages. The two types of behaviour are separated by the critically damped case, $g^2\phi^2 = (3H/2)^2$. Using Eqs. (30) and (16), we can write this critical damping condition in terms of the number of e -folds after critical damping, N_{crit} , as

$$N_{\text{crit}} = \ln\left(\frac{a_{\text{f}}}{a_{\text{crit}}}\right) = \frac{2}{3}\frac{g^2}{\lambda} \simeq \ln\left(\frac{k_{\text{f}}}{k_{\text{crit}}}\right), \quad (34)$$

where subscript “f” refers to the end of inflation and “crit” to the time of critical damping. Wavevectors k_{crit} and k_{f} leave the Hubble radius at t_{crit} and t_{f} , respectively. We see that as g^2/λ increases, cosmological scales are damped like $a^{-3/2}$ during a greater and greater part of inflation. We thus expect that for large enough g^2/λ , the backreaction of the smaller-scale modes will terminate parametric resonance when cosmological-scale $\delta\chi_k$ modes are still greatly suppressed. In other words, there will be a maximum value of g^2/λ for which there is significant amplification of super-Hubble $\delta\chi_k$ perturbations, as anticipated in [17].

We first consider the evolution of the modes which leave the Hubble radius after t_{crit} , *i.e.* $k > k_{\text{crit}}$ (but which are still super-Hubble at the end of inflation, $k < k_{\text{f}}$). These modes are effectively massive during inflation, and hence we can simply use the adiabatic vacuum state, Eq. (17), which for $k \ll aH$ gives

$$|\delta\chi_k(t_{\text{f}})|^2 = \frac{1}{2a_{\text{f}}^3 g \phi_{\text{f}}}. \quad (35)$$

Note that if we define the spectral index n through $\mathcal{P}_{\chi}(k) \propto k^{n-1}$ [24], then for this part of the spectrum we have $n = 4$, an extreme blue tilt.

Next, we will calculate the evolution of modes which leave the Hubble radius before t_{crit} , *i.e.* modes with $k < k_{\text{crit}}$. In this case, the modes are approximately massless when they exit the Hubble radius ($g^2\phi^2 < (3H/2)^2$ for $t < t_{\text{crit}}$), so we can use the standard result for a massless inflaton [29],

$$|\delta\chi_k(t_k)|^2 = \frac{H^2(t_k)}{2k^3}, \quad (36)$$

where t_k is the time that mode $\delta\chi_k$ exits the Hubble radius. We now must use Eq. (33) to evolve the modes during the overdamped period, $t_k < t < t_{\text{crit}}$. Writing $dt = d\phi/\dot{\phi}$, and using the slow-roll approximation $\dot{\phi} \simeq -V_{,\phi}/3H$, we can perform the integral to obtain

$$|\delta\chi_k(t_{\text{crit}})|^2 = \frac{H^2(t_k)}{2k^3} e^{-3F(N_k)}, \quad (37)$$

where N_k is the number of e -folds after time t_k and

$$F(N_k) \equiv N_k - N_{\text{crit}} - \sqrt{N_k} \sqrt{N_k - N_{\text{crit}}} + N_{\text{crit}} \ln \left(\frac{\sqrt{N_k} + \sqrt{N_k - N_{\text{crit}}}}{\sqrt{N_{\text{crit}}}} \right). \quad (38)$$

Next we can readily propagate the modes through the underdamped period, $t_{\text{crit}} < t < t_f$, using Eqs. (32) and (34), giving

$$|\delta\chi_k(t_f)|^2 = \frac{H^2(t_k)}{2k^3} e^{-3F(N_k) - 2g^2/\lambda}. \quad (39)$$

Since the damping term $F(N_k)$ is positive, we see as expected that large-scale modes are strongly damped for large g^2/λ .

Finally, we can approximate the conformal time dependence of all super-Hubble modes during parametric resonance as

$$\delta\chi_k \propto e^{\mu_{\text{max}} x}, \quad (40)$$

if we are near the centre of a resonance band. This is valid since, in this case, the Floquet index μ_k varies only slightly for scales larger than a few times the Hubble radius (*i.e.* the smallest resonant scale) [18].

2. Variances and total resonant growth

Now we can proceed to calculate the field variance, $\langle \delta\chi^2 \rangle$. We will use Eq. (25), restricting the integral to the resonantly growing modes. We begin with the case $g^2/\lambda = 2$. Equation (34) tells us that in this case $N_{\text{crit}} \simeq 4/3$, so that essentially all of the evolution during inflation is in the overdamped regime, and we only need to consider modes with $k < k_{\text{crit}}$. The variance integral will be dominated by modes with $N_k \gg N_{\text{crit}}$, so we may approximate the damping term in Eq. (38) as

$$e^{-3F(N_k)} \simeq \left(\frac{N_{\text{crit}}}{4N_k} \right)^{g^2/\lambda}. \quad (41)$$

For the current case, $g^2/\lambda = 2$, we can now combine the expression (39) with Eqs. (16), (30), (40), and (41) to obtain for the power spectrum on resonant scales at the end of preheating

$$\mathcal{P}_\chi(k, t_e) = \frac{\lambda m_{\text{Pl}}^2}{54\pi^3} e^{2\mu_{\text{max}} x_e}. \quad (42)$$

Here t_e is the time that the resonance shuts down, and x_e is the corresponding scaled conformal time. As we will see, the important thing about this result is that the power spectrum is essentially Harrison-Zel'dovich (independent of k), with spectral index $n = 1$.

We can next rewrite the variance integral, Eq. (25), in terms of the power spectrum as

$$\langle \delta\chi^2(t_e) \rangle = \int_0^{N_0} dN_k \mathcal{P}_\chi(k, t_e) = N_0 \mathcal{P}_\chi(t_e), \quad (43)$$

where $N_0 \simeq 50$ is the total number of e -folds during inflation. Finally, the criterion $g^2 \langle \delta\chi^2(t_e) \rangle \sim \lambda \phi^2(t_e)$ gives, using the value $\phi(t_e) \sim 10^{-2} m_{\text{Pl}}$,

$$\mathcal{P}_\chi(t_e) \sim 10^{-6} m_{\text{Pl}}^2 \quad (44)$$

for the $\delta\chi_k$ power spectrum on cosmological scales at the end of preheating. Note that this result used only the k -independence of the power spectrum (which is a result of the special choice $g^2/\lambda = 2$), and the values of N_0 and $\phi(t_e)$. In particular, the result is independent of λ , unless, contrary to our implicit assumption, λ is so large that $g^2 \langle \delta\chi^2 \rangle > \lambda \phi^2$ already at the start of preheating. In this case, Eq. (44) will be an underestimate.

According to the results from Section II B, we expect synchronization of the other fields to $\delta\chi_k$, so that in particular we expect $\mathcal{P}_\Phi \sim \mathcal{P}_\chi/m_{\text{Pl}}^2$. Therefore we conclude that, for $g^2/\lambda = 2$, the metric perturbation amplitude will indeed be considerably larger than the COBE measured value, even including the effect of backreaction.

Next we will repeat the preceding analysis for the second super-Hubble resonance band, at $g^2/\lambda = 8$. In this case we have $N_{\text{crit}} \simeq 5$, so we must consider modes that exit the Hubble radius both before and after t_{crit} . For the large-scale modes, $k < k_{\text{crit}}$, it will be sufficient to place an upper limit on the variance. Using Eq. (39), but ignoring the damping factor e^{-3F} , we obtain

$$\mathcal{P}_\chi(k, t_f) < \frac{H^2(t_{\text{crit}})}{(2\pi)^2} e^{-2g^2/\lambda} \simeq 2 \times 10^{-8} \lambda m_{\text{Pl}}^2 \quad (45)$$

on scales $k < k_{\text{crit}}$ at the end of inflation. Thus, using Eq. (43), the contribution to the variance from modes with $k < k_{\text{crit}}$ satisfies the (probably very conservative) bound

$$\langle \delta\chi^2(t_f) \rangle_{k < k_{\text{crit}}} < 9 \times 10^{-7} \lambda m_{\text{Pl}}^2. \quad (46)$$

Next we can use Eq. (35) to calculate the contribution to $\langle \delta\chi^2 \rangle$ from smaller-scale modes with $k_{\text{crit}} < k < k_f$,

$$\langle \delta\chi^2(t_f) \rangle_{k_{\text{crit}} < k < k_f} = \frac{1}{16\pi^3 a_f^3 g \phi_f} \int_{k_{\text{crit}}}^{k_f} d^3k \quad (47)$$

$$= \frac{1}{18} \sqrt{\frac{2}{3\pi}} \frac{\lambda \phi_f^5}{g^2 m_{\text{Pl}}^3} \quad (48)$$

$$\simeq 3 \times 10^{-6} \lambda m_{\text{Pl}}^2. \quad (49)$$

Here we have used $k_{\text{crit}}^3 \ll k_f^3$ (which follows from Eq. (34) for $g^2/\lambda = 8$), the relation $k_f/a_f = H(t_f)$, Eq. (30), and the value $\phi_f = 0.2m_{\text{Pl}}$. This value of the small-scale variance exceeds our upper limit on the large-scale variance in Eq. (46), so we can ignore the contribution from the large-scale modes, $\langle \delta\chi^2(t_f) \rangle_{k < k_{\text{crit}}}$.

Now we can again apply the condition $g^2 \langle \delta\chi^2(t_e) \rangle \sim \lambda \phi^2(t_e)$, which in this case gives

$$e^{2\mu_{\text{max}}x_e} \sim 4\lambda^{-1}. \quad (50)$$

Finally, we can use Eq. (39) without approximation to calculate the cosmological-scale power spectrum at the end of preheating, for the case $g^2/\lambda = 8$,

$$\mathcal{P}_\chi(t_e) = \frac{H^2(t_0)}{(2\pi)^2} \exp \left[-3F(N_0) - 2\frac{g^2}{\lambda} + 2\mu_{\text{max}}x_e \right] \quad (51)$$

$$\sim 10^{-14} m_{\text{Pl}}^2. \quad (52)$$

In this case the growth stops before the cosmological perturbations exceed the COBE value, and thus parametric resonance does not change the standard predictions [7] for the size of the fluctuations. Therefore, since the damping of super-Hubble $\delta\chi_k$ modes increases as g^2/λ increases, the standard predictions are not modified for all resonance bands beyond the first, *i.e.* for $g^2/\lambda \geq 8$.

To close this section, consider the behaviour of the large-scale variance if we suppose that inflation started much earlier than the time that cosmological scales left the Hubble radius, *i.e.* let $N_0 \rightarrow \infty$. In this limit we see from the approximation Eq. (41), the expression $H(t_k) \propto N_k$, and Eq. (39) that the variance becomes

$$\langle \delta\chi^2 \rangle_{k < k_{\text{crit}}} \propto \int_{N_{\text{crit}}}^{N_0} dN_k N_k^{2-g^2/\lambda}, \quad (53)$$

which, for $g^2/\lambda = 2$, is divergent as $N_0 \rightarrow \infty$. Such divergences are well-known in inflationary models [30]. Here the divergence suggests that for large enough N_0 , the growth of cosmological-scale modes will stop before they exceed the COBE amplitude, due to the large contribution to the variance from super-cosmological scales. Indeed, for $N_0 \sim 10^6$ (a value not out of the question in chaotic inflation [23]) Eq. (43) gives $\mathcal{P}_\chi(t_e) \sim 10^{-10} m_{\text{Pl}}^2$ for $g^2/\lambda = 2$. For $g^2/\lambda > 3$ the variance converges, although in this case cosmological-scale modes are already suppressed by the mechanism described above.

C. Numerical Results

It is straightforward to check our analytical estimates from the previous section by numerically integrating the coupled set of Hartree approximation evolution equations (20) – (25) and metric perturbation equation (8). We now must evolve a set of modes that fill the relevant resonance band. For example, for $g^2/\lambda = 2$, the first resonance band extends from $\kappa = 0$ to $\kappa = 0.5$ [18]. Again

we begin each mode's evolution inside the Hubble radius during inflation, using the initial vacuum state, Eqs. (17) and (18). Each mode is incorporated into the calculation shortly before it leaves the Hubble radius, so that the spatial gradient terms are never too large. The variances are calculated by performing the discretized integrals, Eqs. (25), only over the resonance band; thus they are convergent. Note that the variances are calculated *simultaneously* with the field backgrounds and perturbations.

In Fig. 3 we present the evolution of the $\delta\chi_k$, $\delta\phi_k$, and Φ_k power spectra on the same cosmological scale as was studied in Section II. All parameters are the same as for Fig. 2, except here we use for the initial background value $\chi(t_0) = 10^{-6} m_{\text{Pl}}$, which means that during preheating $\chi^2 \simeq \langle \delta\chi^2 \rangle$. The evolution is initially similar to that of Fig. 2, only here the growth saturates at $\mathcal{P}_\chi \sim 3 \times 10^{-7} m_{\text{Pl}}^2$, in good agreement with our prediction based on Eq. (44). Also, as expected, the other fields closely follow \mathcal{P}_χ . Whereas in the linear calculations the Einstein constraint equation (10) was satisfied to extremely good accuracy, with the inclusion of backreaction \mathcal{P}_Φ saturates at a factor of roughly 10^3 higher using Eq. (10) than the illustrated result, which used Eq. (8). Note that a similar result was found in [13]. We suspect that this is a fundamental problem related to our attempt to capture some of the nonlinear dynamics with the Hartree approximation. Regardless of which value is used, the cosmological metric perturbations considerably exceed the COBE normalisation. In addition, we find no significant difference in the results when backreaction of metric perturbations is included using Eqs. (28) and (29), as expected if the matter fields dominate the backreaction. Thus all of our presented results exclude the metric backreaction terms.

As discussed above, larger values of g^2/λ result in increased damping of $\delta\chi_k$ on large scales during inflation, and at large enough g^2/λ we expect insignificant amplification of super-Hubble modes. This is illustrated in Fig. 4. Here we examine the second resonance band at $g^2/\lambda = 8$, but use otherwise identical parameters to Fig. 3. Resonance stops at $\mathcal{P}_\chi \sim 10^{-14} m_{\text{Pl}}^2$, consistent with our analytical estimate from Eq. (52), and not exceeding the standard predictions for $\lambda \simeq 10^{-14}$ [7]. Note that the small rise in \mathcal{P}_Φ at late times should not be trusted, as our Hartree approximation scheme will not capture the full nonlinear behaviour. For resonance bands at even higher g^2/λ , we find extremely suppressed cosmological $\delta\chi_k$ amplitudes, in quantitative agreement with the calculations of the previous section.

IV. SELF-INTERACTING χ MODELS

A. Positive coupling

We now consider the addition of a quartic self-interaction term for the χ field, so that our potential becomes

$$V(\phi, \chi) = \frac{\lambda}{4}\phi^4 + \frac{g^2}{2}\phi^2\chi^2 + \frac{\lambda_\chi}{4}\chi^4, \quad (54)$$

with $g^2 > 0$. The significance of such a term for parametric resonance was studied in lattice simulations [27] and analytically [31], but in the absence of metric perturbations. Bassett and Viniegra [17] included metric perturbations, but ignored backreaction. Essentially, for $\lambda_\chi \gtrsim \lambda$ we expect the χ self-interaction to limit the growth of perturbations as compared with the $\lambda_\chi = 0$ case studied above, due to the presence of the “potential wall” $(\lambda_\chi/4)\chi^4$.

More precisely, the linearized equation of motion for the χ field perturbation becomes, with χ self-interaction but ignoring metric perturbations,

$$\ddot{\delta\chi}_k + 3H\dot{\delta\chi}_k + \left(\frac{k^2}{a^2} + 3\lambda_\chi\chi^2 + g^2\phi^2\right)\delta\chi_k + 2g^2\phi\chi\delta\phi_k = 0. \quad (55)$$

Thus for small enough initial χ background, the initial behaviour of the modes will be essentially unchanged from the $\lambda_\chi = 0$ case. However, when the χ background grows to the point that $\chi^2/\phi^2 \gtrsim g^2/\lambda_\chi$, the analytical parametric resonance theory of Section II A no longer applies, and we may expect the perturbations to stop growing. Since, as discussed above, for the significant production of super-Hubble modes we require $g^2 \simeq \lambda$, we expect that χ self-interaction will shut down the resonance when $\chi^2/\phi^2 \sim \lambda/\lambda_\chi$, as long as $\lambda_\chi \gtrsim \lambda$. If $\lambda_\chi < \lambda$, then the χ^4 interaction term will not lead to a shutdown of the resonance since (based on our numerical simulations) the homogeneous χ field never substantially exceeds the value of the inflaton background.

We have confirmed this expectation numerically, and we give an example of our results in Fig. 5. Here we have included Hartree backreaction and metric perturbations, and used coupling constant values $\lambda = 10^{-14}$, $g^2/\lambda = 2$, and $\lambda_\chi = 10^{-10}$, and initial backgrounds $\phi(t_0) = 4m_{\text{Pl}}$ and $\chi(t_0) = 10^{-6}m_{\text{Pl}}$. We indeed observe the termination of the super-Hubble modes’ growth at approximately the time when $\chi^2/\phi^2 = \lambda/\lambda_\chi$.

Note that, *in the absence of backreaction*, Bassett and Viniegra observed a continued slow growth of super-Hubble perturbations after the initial termination of the resonance when $\chi^2/\phi^2 \sim \lambda/\lambda_\chi$ [17]. We confirmed this result; however, we note that when we include the backreaction term $3\lambda_\chi\langle\delta\chi^2\rangle$ in the evolution equations, we expect backreaction to become important also at the time that $\chi^2/\phi^2 \sim \lambda/\lambda_\chi$, with our choice $\chi^2 \simeq \langle\delta\chi^2\rangle$. Hence,

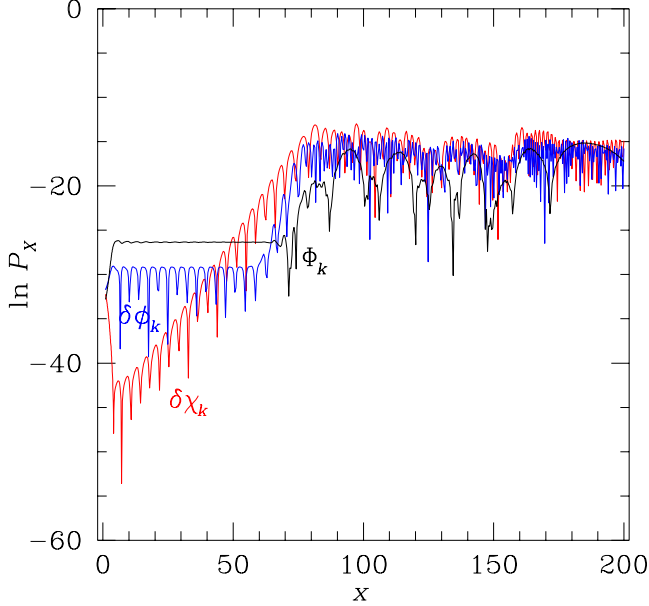


FIG. 3. Numerical simulation of cosmological-scale perturbations with Hartree backreaction terms included and with a non-zero initial background, $\chi(t_0) = 10^{-6}m_{\text{Pl}}$. Plotted are the logarithms of the power spectra $\mathcal{P}_X(k) = (k^3/2\pi^2)|X_k|^2$, for $X_k = \delta\chi_k$, $\delta\phi_k$, and Φ_k , using $m_{\text{Pl}} = 1$, $g^2/\lambda = 2$, and $\lambda = 10^{-14}$. Backreaction terminates the growth of each field perturbation.

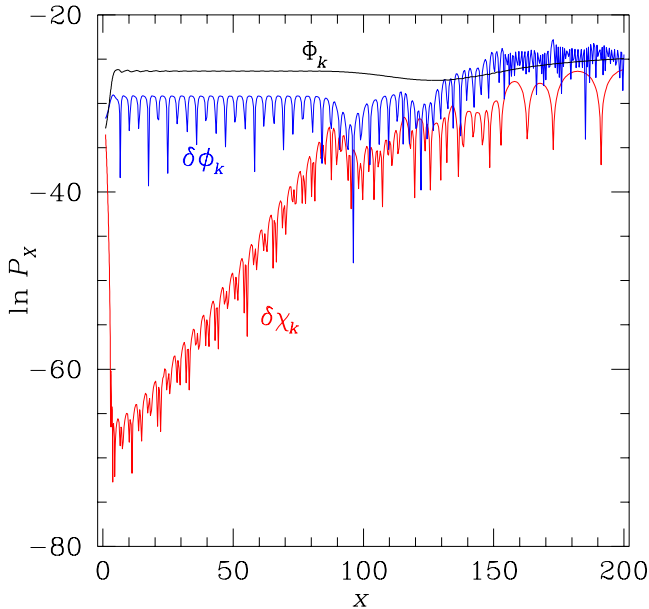


FIG. 4. Same as Fig. 3, except for parameters lying in the second resonance band, *i.e.* $g^2/\lambda = 8$, and with $\chi(t_0) = 10^{-6}m_{\text{Pl}}$ and $\lambda = 10^{-14}$. Here backreaction of the small-scale modes terminates the growth before the COBE value is exceeded.

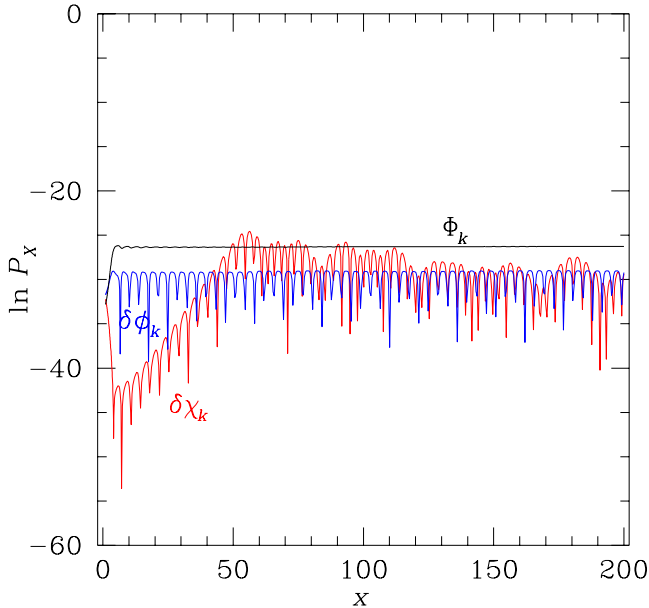


FIG. 5. Same as for Fig. 3, except with χ field self-coupling $\lambda_\chi = 10^{-10}$. The χ self-coupling causes the early termination of growth.

as seen in Fig. 5, the slow growth is completely suppressed.

B. Negative coupling

The presence of χ self-coupling means that we no longer require $g^2 \geq 0$ for global stability. In fact, for the case $g^2 < 0$, the potential will be bounded from below for $\lambda\lambda_\chi/g^4 > 1$ [32]. This *negative coupling* case was studied in the absence of metric perturbations using lattice simulations in [32], and without backreaction in [16]. The behaviour of the fields is qualitatively different in the negative and positive coupling cases. For $g^2 < 0$, potential minima exist with non-zero homogeneous part of the χ field. Thus, assuming the fields fall into these minima, the problem of choice of χ background discussed in previous sections for the positive coupling case is alleviated.

For initial homogeneous χ fields large enough ($\chi(t_0) \gtrsim m_{\text{Pl}}$), we find numerically that the fields fall into the potential minimum by the end of inflation, and the two fields subsequently evolve in step during preheating. This effectively reduces the system to a single-field system, and hence no resonance is possible on super-Hubble scales.

To see this explicitly consider the case $\lambda_\chi = \lambda$, for which the symmetry of the potential requires the potential minima to lie along $\chi^2 = \phi^2$. If we choose the same initial signs for χ and ϕ , then during preheating the backgrounds lie along the attractor $\phi = \chi$. Similarly,

since the behaviour of super-Hubble modes is essentially the same as that of the backgrounds, we have $\delta\chi = \delta\phi$ during preheating. Then the perturbation equation (55) becomes

$$\ddot{\delta\chi_k} + 3H\dot{\delta\chi_k} + \left[\frac{k^2}{a^2} + 3(\lambda + g^2)\phi^2\right]\delta\chi_k = 0. \quad (56)$$

Thus the effective mass of the $\delta\chi$ oscillations is precisely three times the effective mass of the background inflaton oscillations (*cf* Eq. (3)), so that just as with the case of the inflaton perturbations in Eq. (14), there will be no resonance on super-Hubble scales for all allowed values of g^2 . We have confirmed this numerically; indeed more generally, as long as initially $\chi(t_0) \sim m_{\text{Pl}}$ but for any $\lambda_\chi \geq \lambda$, the two fields will be proportional during preheating and no super-Hubble resonance will result.

This result assumes that during preheating only the “field” $\delta\chi + \delta\phi$ is excited. If orthogonal field excitations $\delta\chi - \delta\phi$ are present, they can grow resonantly. The effective squared mass of $\delta\chi - \delta\phi$ excitations is $(3\lambda - g^2)\phi^2$, so that according to the analytical parametric resonance theory of Section II A, super-Hubble resonance will occur near $3\lambda - g^2 = 2n^2(\lambda + g^2)$, for integral n (we require $n \geq 2$ for negative g^2). That is, super-Hubble $\delta\chi - \delta\phi$ modes will grow for $g^2 \simeq \lambda(3 - 2n^2)/(1 + 2n^2)$. However, numerically we observe only extremely small components $\delta\chi - \delta\phi$ by the end of inflation, so their growth is substantially delayed.

On the other hand, for small initial homogeneous part $\chi(t_0) \ll m_{\text{Pl}}$, we find that the potential minima are not reached by the end of inflation, and the two fields evolve in a very complicated manner during preheating. The analytical theory of parametric resonance cannot be applied, but numerically we do find roughly exponential growth of super-Hubble modes in this case, as found in [16]. The growth rate increases as g^2 decreases towards the value at which global instability sets in, $g^2 = -\sqrt{\lambda\lambda_\chi}$.

We have illustrated this case in Fig. 6, using the parameter values $\lambda = 10^{-14}$, $g^2 = -0.5\lambda$, $\lambda_\chi = \lambda$, $\phi(t_0) = 4m_{\text{Pl}}$, and $\chi(t_0) = 10^{-6}m_{\text{Pl}}$. Here the growth rates and final power spectra values are comparable to the $\lambda_\chi = 0$ case of Fig. 3, though the $\delta\chi$ field is not damped during inflation for negative coupling. For $\lambda_\chi > \lambda$, the growth is terminated early, just as in the positive coupling case.

V. SUMMARY AND DISCUSSION

In this paper we have studied backreaction effects on the growth of super-Hubble cosmological fluctuations in a specific class of two field models with a massless inflaton ϕ coupled to a scalar field χ . Our study was based on the Hartree approximation.

For the non-self-coupled χ field case, we found that backreaction has a crucial effect in determining the final

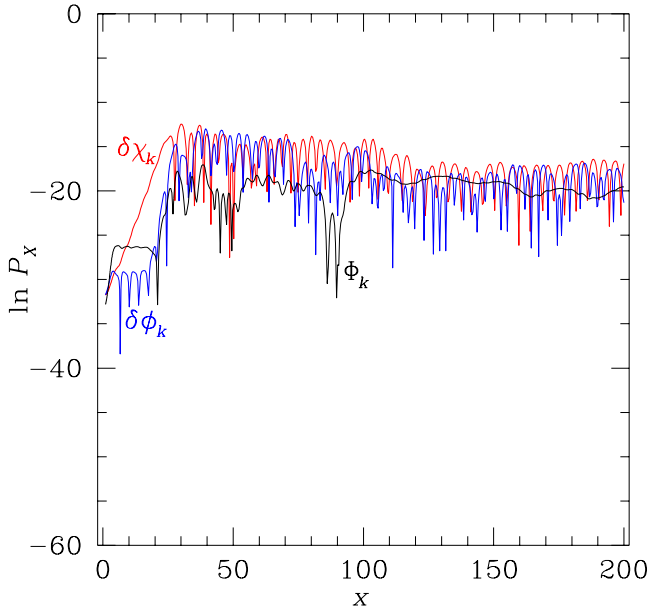


FIG. 6. Same as for Fig. 5, except for the negative coupling case, with parameters $\lambda = 10^{-14}$, $g^2 = -0.5\lambda$, and $\lambda_\chi = \lambda$. Here the growth is comparable to the $\lambda_\chi = 0$ case of Fig. 3, even though analytical parametric resonance theory cannot be applied here.

amplitude of fluctuations after preheating. For values of the coupling constants satisfying $g^2/\lambda = 2$ (the ratio predicted in supersymmetric models), the predicted amplitude of the super-Hubble metric perturbations at the end of preheating is too large to be consistent with the COBE normalization, thus apparently ruling out such models. One possible loophole is the backreaction contribution from super-cosmological scale fluctuations. For sufficiently long periods of inflation, the predicted amplitude can be consistent with the COBE normalization. In addition, the final amplitude of the fluctuation spectrum is independent of the coupling constant λ . Note that the growth of inflaton fluctuations $\delta\phi_k$ (and hence metric perturbations Φ_k) occurs in these models either through coupling to $\delta\chi_k$ via a homogeneous background χ field or through nonlinear evolution effects.

The situation for $g^2/\lambda \gg 1$ is very similar to the previously studied case of a massive inflaton in the broad resonance regime [12–14]. Cosmological-scale $\delta\chi_k$ modes are significantly damped during inflation, and the end of resonant growth is determined by the growing small-scale modes. Already for the second resonance band (centred at $g^2/\lambda = 8$) cosmological metric perturbations are not amplified above the COBE normalization value. This implies that preheating does not alter the standard predictions for the Φ_k normalization in $(\lambda/4)\phi^4$ inflation for the second and all higher resonance bands. The important difference between the model we have studied and the massive inflaton case is that, in the massive model, weak super-Hubble suppression at small g^2 is accompa-

nied by weak resonant growth during preheating [13], so that no significant super-Hubble amplification is possible.

The inclusion of χ field self-interaction alters the evolution in a predictable way: the resonant growth stops when $\chi^2/\phi^2 \sim \lambda/\lambda_\chi$, as long as $\lambda_\chi \gtrsim \lambda$. This means that we are unable to rule out models (on the basis of a too large production of metric perturbations) with $\lambda_\chi/\lambda \gtrsim 10^4$. In the negative coupling case, there are two possibilities. For large initial χ backgrounds, $\chi(t_0) \sim m_{\text{Pl}}$, the system becomes essentially single-field, and no resonance occurs (at least until late times). For small initial χ , exponential growth occurs for large enough allowed $|g^2|$.

The Hartree approximation provides a useful approach for the inclusion of the effects of backreaction. However, as mentioned in Section III, this approximation misses terms which could contribute to the evolution of fluctuations in an important way. We believe, nevertheless, that our results are sufficiently accurate to predict, for the models studied, whether or not the metric perturbation amplitude after preheating is consistent with the COBE measurement. Nonlinear effects, or rescattering, will primarily affect the detailed evolution of matter fields after backreaction is important. Still, it is of great interest to extend our analysis to a full nonlinear treatment, as was done in the absence of gravitational fluctuations in [33,28,27], and including metric fluctuations in [9] for single field models.

ACKNOWLEDGMENTS

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